Practice Problems (Mapping)

1. Let $A = \{a, b, c\}$ and $B = \{1, 2\}$.

(a) How many $1-1$ mappings are there from A to A? List them.

(b) How many mappings are there from A onto A?

(c) How many $1-1$ mappings are there from A to B?

(d) How many mappings are there from A onto B? (HINT: It may be easier to count the mappings that are not onto.)

(e) How many $1-1$ mappings are there from B to A?

(f) How many mappings are there from B onto A?

2. Explain why each of the following is not a function.

(a) $\alpha : \mathbf{R} \to \mathbf{R}$ defined by

$$\alpha(x) = \frac{x}{x^2 - 4}$$

for every $x \in \mathbf{R}$.

(b) $\beta : \mathbf{R} \to \mathbf{R}$ defined by $\beta(x) = x \ln |x|$ for every $x \in \mathbf{R}$.

(c) $\gamma : \mathbf{Q} \to \mathbf{Q}$ defined as follows: For a rational number r, write $r = a/b$, where a and b are integers and $b \neq 0$. Set

$$\gamma(r) = \gamma(\frac{a}{b}) = \frac{a+b}{a^2+b^2}.$$

4. Define $\alpha : \mathbf{R} \to \mathbf{R}$ by $\alpha(x) = 3x + 5$ for all $x \in \mathbf{R}$.

(a) Prove that α is $1-1$.

(b) Prove that α maps $\mathbf{R}$ onto $\mathbf{R}$.

5. Define $\beta : \mathbf{R} \to \mathbf{R}$ by $\beta(x) = 3x^2 + 5$ for every $x \in \mathbf{R}$. Prove that β is neither $1-1$ nor onto.

6. Let $A = \mathbf{R} - \{3\}$ and $B = \mathbf{R} - \{2\}$ and define $\gamma : A \to B$ by $\gamma(x) = \dfrac{2x+1}{x-3}$.

(a) Verify that γ maps A to B; that is, show that for all $a \in A$, $\gamma(a) \neq 2$. [HINT: Use contradiction.]

(b) Prove that γ is $1-1$.

(c) Prove that γ maps A onto B.